The history and development of numerical analysis in Scotland: a personal perspective*

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Abstract

An account is given of the history and development of numerical analysis in Scotland. This covers, in particular, activity in Edinburgh in the first half of the 20th century, the collaboration between Edinburgh and St Andrews in the 1960s, and the role played by Dundee from the 1970s. I will give some reminiscences from my own time at both Edinburgh and Dundee.

1 Introduction

To provide a historical account of numerical analysis (or of anything else), it is necessary to decide where to begin. If numerical analysis is defined to be the study of algorithms for the problems of continuous mathematics [16], then of course it has a very long history (see, for example, [6], [13]). But "modern" numerical analysis is inextricably linked with computing machines. It is usually associated with programmable electronic computers, and is often said to have its origins in the 1947 paper by von Neumann and Goldstine [10]. The name apparently was first used around that time, and was given formal recognition in the setting up of the Institute for Numerical Analysis, located on the campus of the University of California at Los Angeles [3]. This was a section of the National Applied Mathematics Laboratories of the National Bureau of Standards, headed by J. H. Curtiss, who is often given credit for the name.

Others consider modern numerical analysis to go back further than this; for example Todd [15] suggests it begins around 1936, and cites papers by Comrie and Turing. But if what is required is a systematic study of what we now think of as numerical analysis, in conjunction with the use of calculating machines, then the origin of numerical analysis in Scotland can be traced back even further, to the early years of the 20th century.

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2 Edinburgh: Early Years

To be specific, the story begins in Edinburgh, with the arrival of E. T. Whittaker (1873–1956). From a Lancashire family, Whittaker won a Scholarship to Trinity College, Cambridge in 1892. He graduated in Mathematics, and he then taught at Cambridge where he developed an interest in Astronomy. This led in due course to his appointment as Astronomer Royal of Ireland in 1906, and at the same time he became Professor of Astronomy at the University of Dublin. However, on the death of George Chrystal, he came to the University of Edinburgh in 1912 to fill the Chair of Mathematics. In 1913, he opened a Mathematical Laboratory in Edinburgh where students were trained to do machine calculations. Subjects to be taught included interpolation, difference formulae, determinants, linear equations, the numerical solution of algebraic equations, numerical integration, least squares and the numerical solution of differential equations. When the list of topics became known, it generated such interest that the Edinburgh Mathematical Society organized a colloquium in the summer of 1913 with the primary aim of providing instruction in aspects of numerical mathematics.

E. T. Whittaker in 1930

Of course, the association of Edinburgh with calculating machines can be traced back further than that, at least to John Napier (1550 – 1617), who was born in Edinburgh. He was the inventor of logarithms and "Napier's Bones" (numbering rods made of ivory, used for mechanically multiplying, dividing and taking square roots and cube roots) [9].

But the arrival of Whittaker in Edinburgh saw various branches of what we now think of as numerical analysis taught in a systematic way, probably for the first time in any British University. David Gibb was one of the two lecturers on the staff in Edinburgh when Whittaker arrived, and in 1915 he published a book [5] based on some of this material in the Edinburgh Mathematical Tracts series (edited by Whittaker). In a "Preliminary note on computation", he says: "Each desk [in the Mathematical Laboratory] is supplied with a copy of Barlow's Tables,... a copy of Crelle's multiplication table (which gives at sight the product of any two numbers each less than 1000) and with tables giving the values of the trigonometric functions and logarithms. These may ... be supplemented by ... any of the various calculating machines now in use ... Success in computation

depends partly on the proper choice of a formula and partly on a neat and methodical arrangement of the work." Apparently, when asked by a girl student: "Sir, what is the formula for iteration?", Gibb replied "There is no formula; you just iterate."

George Robinson was also a member of Whittaker's staff in Edinburgh, and the book by Whittaker and Robinson [19], first published in 1924, is a collection of the material taught in the Laboratory. In the preface it says: "The present volume represents courses of lectures given at different times during the years 1913 - 1923 by Professor Whittaker to undergraduate and postgraduate students in the Mathematical Laboratory of the University of Edinburgh, and may be regarded as a manual of teaching and practice of the Laboratory."

A. C. Aitken (1895–1967) was born and educated in New Zealand and came to Edinburgh in 1923 to study for a PhD with Whittaker. In 1925 he was appointed to a lectureship in Actuarial Mathematics, in due course became Reader in Statistics, and in 1946 succeeded Whittaker to the Chair in Mathematics, a post he held till his retirement in 1965. According to Tee [14], Whittaker was the only person in the UK teaching determinants and matrices when Aitken arrived in Edinburgh. Aitken became interested in this area, and applied matrix algebra to numerical analysis and statistics. What we now think of as Householder matrices appeared in Aitken's book with H. W. Turnbull, Professor of Mathematics at St Andrews [17]. Aitken also published papers on polynomial equations and eigenvalue problems: the QD algorithm developed by Heinz Rutishauser in the 1950s essentially generalizes some of his results and turns them into an effective algorithm. The acceleration process which bears his name is still used today, for example in applications to the Schwarz method for domain decomposition [4], [8]).

A. C. Aitken

An honours course entitled Mathematical Laboratory existed from about 1920 to 1960. The name was changed to Numerical Analysis from the academic year 1960-61, so this was the first time in Edinburgh that this name was used for an honours course. Aitken took over the teaching from Whittaker about 1946 (although he may have contributed before that time), and continued to teach it until 1961. The Mathematics Department had lots of hand calculating machines; for the benefit of the students, Aitken would appear to operate one of the machines, but would actually do the calculations in his head. The change of name may have been at the instigation of James Fulton, who took over the

teaching of the course in 1961-62, as Aitken's health was giving cause for concern.

In 1961, the University Grants Committee had a funding round for equipment, covering all Universities in the UK. When contacted by the Committee, the Administration in Edinburgh referred this to Aitken, who did not see any requirement for a computer. Nevertheless, Edinburgh University set up a Computer Unit in 1963 with Sidney Michaelson (1925–1991) as Director. Michaelson had been appointed to a Lectureship in Mathematics at Imperial College in London in 1949, and he worked on numerical analysis throughout the 1950s and early 1960s, but his main interest was in the design and construction of digital computers. Working with K. D. Tocher, in the early 1950s he built a machine (ICCE1), using cheap war-surplus parts, which had a novel modular design. By 1957, Michelson and Tocher were well on the way towards the construction of a valve machine, when funding was almost completely cut off: Imperial College thought that the successful development of this machine would inhibit the acquisition of a new Ferranti Mercury computer.

In any event, Michaelson moved to Edinburgh. He brought with him M. R. Osborne as Assistant Director of the Computer Unit. Osborne was born in 1934 in Australia. After a BA degree in Melbourne, he was employed from 1957 as a Scientific Officer for the Royal Australian Navy. He was posted to the UK where he spent some time in the Mathematics Division of the Admiralty Research Laboratory (ARL). He then had spells as Lecturer at the University of Reading and also at Imperial College, during which time he completed a PhD at Imperial College, supervised by Michaelson. Osborne was mainly interested at that time in finite difference methods for both ordinary and partial differential equations. He was joined in the Computer Unit by Donald Kershaw. Kershaw became interested in numerical analysis when he joined the Mathematics Division at ARL in 1957, after two years with Vickers-Armstrong (Aircraft) in Weybridge. Following a period at ARL (where for a time he shared an office with Osborne), he moved to Edinburgh in March 1964 to teach numerical analysis. Kershaw's main interests were differential and integral equations.

Michaelson set up a landline link from the Computer Unit to the University of Manchester Atlas Computer, which came into general use in 1963. The link was largely the result of a Manchester initiative, and was accepted by Edinburgh University following protests (mainly by chemists) about the failure to acquire a computer in the 1961 funding round. When a Post Office technician came to install the landline in the Computer Unit, he was unsure how to proceed, so Michaelson drew a circuit diagram on the spot and correctly installed the connections while the technician watched. Edinburgh was allocated 15 minutes computing time per day; programmes were written in Atlas Autocode and punched on to paper tape.

The Manchester Atlas Console

3 How I became a numerical analyst

There was no tradition of University attendance in my family, but as I progressed through school and passed various examinations, it became clear that this would be possible. My main interests were in Science and Mathematics, and I decided I wanted to study engineering, with the intention of becoming an aeronautical engineer. When I told my Science teacher this, he gave me the only piece of career advice I ever got: "Engineers are ten a penny; do pure science." Whether this was good advice or not, I will never know, but I took it and entered the University of Edinburgh in October 1960 to take a BSc in Physics. Of course, I also studied Mathematics, and as a first year student attended lectures by Aitken on the material in his book "Determinants and Matrices" [1]. He was always keen to highlight his prodigious memory, and some time in his lectures was usually set aside for tricks and demonstrations. But, as mentioned above, his health was failing.

As an aside, I last saw Aitken when I attended a public lecture he gave (maybe in 1964) where he argued the case for a duodecimal system for currency rather than the decimal system at that time being proposed for the UK. I remember him saying (so he probably said it more than once): "They scratch, but we punch!" first making scratching motions with his 10 fingers and thumbs and then pointing to the 12 knuckles on the fingers of one hand.

After two years studying, I decided I preferred Mathematics to Physics and switched to a degree in Mathematical Sciences. In my third year, in early 1963, I could choose between a course on Numerical Analysis and one on Statistics, and having tried both for a while, decided I did not like Statistics much and continued with the Numerical Analysis option. Topics covered included rounding errors, interpolation, orthogonal expansions, Fourier and Chebyshev series, finite differences, various difference operators, numerical integration, initial value problems for ordinary differential equations, iterative methods for solving equations, the power method for eigenvalue problems, and Gaussian elimination. In addition to large books of multiplication tables, which when opened, measured about two feet by three feet, various calculating machines were available, mainly Facits and Brunsvigas, hand operated. There were also some electric calculators. The recommended book was by R. A. Buckingham [2].

Brunsviga (left) and Facit

I took the Numerical Analysis course in both my third and fourth years. It was taught mainly by Fulton, but in 1964, he decided to make use of the expertise available in the Computer Unit and so it contained some lectures by Osborne on the numerical solution of boundary value problems in ordinary differential equations. Also, least squares problems and harmonic analysis were treated.

During my fourth year, I attended some job interviews and accepted a job with ICI in Billingham. After my graduation ceremony in Edinburgh in July 1964, I was speaking with a class mate, Alex Wight, who told me he had a summer job in the Edinburgh University Computer Unit, and suggested I go round to see the place. Of course, there was no computer there at the time, so what there was to see was not immediately obvious. But I was introduced to Osborne, and at some point he said there was a vacancy for a Demonstrator and indicated that if I was interested, I could likely get it, with the opportunity to do an MSc. The possibility of something like this had never previously crossed my mind, and of course I already had a job. However, it was an attractive idea, the salary was the same as being offered by ICI, so I thought it over, applied, and was duly offered the job. Before accepting, I wrote to tell ICI and suggested joining them later on, but they said just to apply to them again, which of course I never did.

So I started work in the Edinburgh University Computer Unit in October 1964. I attended lectures on computing and numerical methods, given as part of a postgraduate diploma course started that year, and learned to write programmes in Atlas Autocode. Part of my job included examining output when it arrived back from Manchester, and helping users to identify programming errors. Unfortunately the link was somewhat unreliable, and for a time ordinary mail was used to send programs. So it took maybe three days to find out that a comma or something had been omitted from the program, causing it to fail: of course great care was taken to avoid trivial errors!

I also embarked on research with Osborne. He had widened his interests to include Chebyshev approximation, and he asked me to look at linear Chebyshev approximation problems, and the way in which linear programming could be used, as part of an MSc project. I wrote programmes in Atlas Autocode to run on the Manchester Atlas. But I also recall on one occasion travelling through to Glasgow to run some programmes on the KDF9 there, for which an Atlas Autocode compiler had been written.

A DEC PDP/8 was installed in the Computer Unit in late 1965 and caused great excitement.

4 St Andrews

A. R. Mitchell (1921–2007) was born in Dundee. He was educated at Morgan Academy, and in 1938 went to University College Dundee, at that time a college of the University of St Andrews, to study Mathematics. Partly due to the war, student numbers were low, and he was the only Honours student in Mathematics. On graduating in 1942, he was called up for military service and went to the wartime Ministry of Aircraft Production in London, where he remained until the end of the war. His duties included the interrogation of captured Luftwaffe pilots; some years later he met one of them at a conference.

In 1946, Mitchell decided he would like to do a PhD, and returned to Dundee to see if this was possible. There was no available supervisor in Dundee, but he made contact with D. E. Rutherford in St Andrews, who agreed to act as supervisor if Mitchell would become an Assistant Lecturer for the duration of his PhD. His thesis was concerned with relaxation methods in compressible flow (Rutherford's main research interest was Lattice Theory, so the supervision must have been fairly nominal), but he developed an interest in numerical analysis, initially as a means of tackling fluid dynamics problems using Southwell's relaxation methods.

On completion of his PhD in 1950, Mitchell remained at St Andrews as a Lecturer. In 1953, J. D. Murray became his first PhD student, working on a topic in boundary layer fluid dynamics, and his second was J. Y. Thompson, who started in 1954 working on numerical aspects of fluid dynamics. J. D. Lambert was a student at St Andrews and a member of the Honours class in 1953–54 when Mitchell taught a course in numerical analysis, the first time numerical analysis had been taught in St Andrews. After graduating, Lambert worked at the Admiralty Research Laboratory (in the Fluid Dynamics Group) from 1954–1957, before having short spells at Memorial University of Newfoundland in Canada and Magee University College, Londonderry, Northern Ireland. He returned to St Andrews as a Lecturer in 1959, and became Mitchell's third PhD student, working on an idea of Mitchell's of incorporating higher derivatives into methods for ODEs. Other PhD students who came in the 1960s were Graeme Fairweather, Sandy Gourlay, Pat Keast (who had been in my class at Edinburgh), John Morris (all supervised by Mitchell), and Brian Shaw (who was supervised by Lambert).

In 1964, St Andrews acquired an IBM 1620 Model II. Apparently this was capable of solving Laplace's equation in a cube using an optimal alternating direction finite difference method with a $5 \times 5 \times 5$ mesh in 15 minutes; in a square, a 20×20 mesh could be tackled. The Computer was housed in the Observatory, over a mile from the Department of Mathematics, and hands-on access was provided for an hour each morning and afternoon, with no exceptions, even when the printer ribbon wrapped itself round the type bar, a frequent occurrence. Batch jobs could run at other times.

In 1965, Mitchell started going to evening classes in Dundee to learn Russian. He was then able to keep up with the Russian literature and was perhaps one of the first in the West to appreciate the importance of the work being done in the USSR on high order difference methods for PDEs, in particular by Samarskii, Andreyev and D'Yakonov. He met D'Yakonov at the ICM meeting in Moscow in 1966, and as a consequence, the latter visited Mitchell a few years later. A byproduct was that many others in the West became much more aware of the activity in the USSR concerning split operator techniques.

5 Collaboration between St Andrews and Edinburgh

A numerical analysis course was given in the University of Aberdeen in 1958–59 by F. W. Ponting. This was the first in Aberdeen, and was based on material from the book by D. R. Hartree [7], maybe the first book on numerical analysis to use the name. Some numerical analysis was apparently taught in the University of Glasgow in the early 1960s, using the English Electric DEUCE and then a KDF9: Glasgow was the first University in Scotland to have an electronic computer, when the DEUCE was installed in 1958. But the main centres for numerical analysis in Scotland by the mid 1960s were undoubtedly Edinburgh and St Andrews. In particular Osborne and Mitchell were keen for more interaction between their research groups and it was Osborne who initiated the idea of a conference. A detailed history of the origins of that conference are given in [18], and so a full account will not be given here. But about 25 people met in June 1965 in St Andrews for two (or maybe three) days, with Mitchell (the main organizer) and the other three members of the organizing committee (Osborne, Kershaw and Lambert) giving the main talks. I still have a folder (unfortunately now empty of its original contents) with "Symposium on Solution of DEs, St Andrews, June 1965" written on it. Although intended mainly for the two Scottish groups, a number of English based numerical analysts attended, with John Mason from Oxford probably travelling furthest.

A. R. Mitchell (left) and M. R. Osborne in Dundee in 1997

A one-day meeting on Chebyshev approximation was also organized by Osborne in Edinburgh, which was attended by the St Andrews group and others from as far afield as London. There were four speakers: Osborne and I gave talks, and also M. J. D. Powell and A. R. Curtis, both of whom had been invited up from Harwell. It was the first talk I had ever given, I was in very distinguished company, and I remember it as a rather traumatic experience.

Osborne left Edinburgh for Australia at the end of 1965, to become Director of the Computer Centre at the Australian National University, and I followed him there to start a PhD about four months later. Also in late 1965 Lambert moved from St Andrews

to Aberdeen. However, in 1967 Mitchell organized a second meeting in St Andrews. This was held from 26-30 June, and called "Colloquium on the Numerical Solution of Differential Equations". There were 85 participants, 18 main speakers, each giving a 50 minute talk, and 19 shorter talks. Two of the main speakers were from overseas, but it was very much a UK event.

Incidentally when I arrived at ANU in June 1966, an IBM 360/50 had just been installed. The first thing I did was learn a new programming language, PL/1, which had just been developed by IBM. The name was originally NPL (New Progamming Language) but was changed to avoid confusion with the National Physical Laboratory. Programmes were put onto punched cards. At first the machine was only generally available during office hours. But I was able to go into the Computer Centre in the evening, switch it on, get it up and running (I recall this included mounting large disks on disk drives), put my deck of cards in a hopper, run my programme and print the output. After a few runs, I would switch everything off and go home. Of course usage grew rapidly, normal hours were extended, and this experience of "personal computing" did not last long! Nor did PL/1 at ANU, because Osborne decided after a visit to Europe that it should be replaced, and so we all started to use Fortran.

6 Dundee

D. S. Jones was appointed to the Ivory Chair of Mathematics in Queen's College, Dundee in 1965. Queen's College was part of the University of St Andrews, although as a consequence of the report of the Committee on Higher Education produced under the Chairmanship of Lord Robbins in 1963, the processes necessary to establish Dundee as a separate University were well under way.

Jones had the foresight to see numerical analysis as a growth area. He started up an MSc in Numerical Analysis and Programming, taught mainly by R. P. Pearce, the only staff member at Dundee at the time who could be considered a numerical analyst: Pearce had collaborated with Mitchell in the early 1960s (three joint papers were published in 1962 and 1963 on finite difference methods).

Queen's College separated from St Andrews to become the University of Dundee on 1st August 1967. Jones had obtained approval for a Chair of Numerical Analysis in Dundee, and also in 1967 Mitchell (then a Reader at St Andrews) was appointed. Mitchell was joined in Dundee by Lambert and Gourlay as Senior Lecturer and Lecturer respectively, and research students and postdoctoral research fellows were attracted, the latter largely through funding from NCR (The National Cash Register Company) and the MoD (Ministry of Defence). Despite losing Mitchell and his group, St Andrews brought in replacements to maintain a presence in numerical analysis. In particular, G. M. Phillips, who worked mainly in Approximation Theory, was appointed from Southampton University in 1967. Other numerical analysts who came to St Andrews then or later on were M. A. Wolfe, J. H. McCabe and G. E. Bell.

But with the Edinburgh group diminished, the appointment of Mitchell resulted in the centre of gravity of numerical analysis in Scotland clearly shifting to Dundee. I arrived to take up an MoD Fellowship in September 1969, just missing the third conference, a "Conference on the Numerical Solution of Differential Equations", the first such meeting to be held in Dundee. It attracted 148 participants, with 8 invited speakers all from

overseas, and around 45 others from outside the UK. So this was the first conference with a truly international flavour.

Among others to hold Research Fellowships in Dundee by 1970 were John Morris, Sean McKee and Nancy Nicholls. Traffic was not all one way, of course. Pearce had left in 1967 for Imperial College (he was eventually to be appointed to a Chair in Meteorology in Reading), and Gourlay left in 1970 to work for IBM, although this was a good move for me, as I was appointed to the vacant Lectureship.

The academic year 1970-71 was a special one for numerical analysis in Dundee, and really put Dundee (and Scotland) on the numerical analysis map. Mitchell obtained funding from the UK Science Research Council for a Numerical Analysis Year in Dundee. The aim was to promote the theory of numerical methods and to upgrade the study of numerical analysis in British universities and technical colleges. This was done by arranging lecture courses, seminars and conferences in Dundee so that workers in the field would have the opportunity to hear about and to discuss recent research. Some 34 of the world's leading numerical analysts visited Dundee during this period, some for short periods and others for longer periods up to the full year. Of course I enjoyed being in the company of so many big names.

Five conferences were held in Dundee during that year. As well as three smaller meetings, there was a "Conference on Applications of Numerical Analysis" held from 23–26 March, 1971, with 170 participants, organized by Morris, by now a Lecturer in Dundee. I also organized a "Conference on Numerical Methods for Nonlinear Optimization" which was held from 28 June–1 July, 1971, with 198 participants. One of the main speakers at the March meeting was Lothar Collatz from Hamburg, making the first of many visits to Dundee. Another was G. H. Golub, who first came to Dundee in 1970, and who also returned many times. Indeed their enthusiasm and support for these and subsequent meetings played a large part in attracting participants from outside the UK. Among the UK invited speakers at the March meeting were K. W. Morton and M. J. D. Powell, both of whom had also been invited to the 1967 St Andrews meeting. They went on to be regular participants (not just as invited speakers), and the series benefitted greatly from the continuing presence of numerical analysts such as these.

In 1973, a "Conference on the Numerical Solution of Differential Equations" was held from July 3–6, organized by Morris who had taken over as the main conference organizer. There were 234 participants, with 20 invited speakers, and 43 submitted papers presented in parallel sessions. I edited the Proceedings (of the invited talks), and said in the Preface: "This was the 5th in a series of biennial conferences in numerical analysis, originating in St Andrews University, and held in Dundee since 1969". So this was perhaps the first explicit acknowledgement of the numbering system, with the March 1971 meeting probably interpreted as the fourth in the series.

Also in 1973, Roger Fletcher moved from Harwell to Dundee. He was (and continues to be) a leading figure in optimization, and this was a major strengthening of numerical analysis in Scotland, and of the numerical analysis group in Dundee. It also represented a significant broadening of the numerical analysis base, and this was reflected in the fact that the 1975 meeting was a "Conference on Numerical Analysis", a name which was retained. About 200 people attended the meeting from July 1-4, with 16 invited talks and 45 contributed talks.

So the biennial series was now well into its stride, and with a fairly well established pattern. It survived the departure from Dundee of Morris, who moved to Waterloo in

1975, leaving me to shoulder the main organizational load. D. F. Griffiths, who was appointed to Dundee in 1970, had shifted his interests to numerical analysis, and started to work with Mitchell, in particular on finite element methods. Mitchell's interest in finite element methods had begun in the late 1960s, apparently prompted by the arrival of Dick Wait as a PhD student, as he announced that he would like to do a PhD in that area. Griffiths organized the 1983 meeting when I was on sabbatical leave in Australia and New Zealand, and from 1985 onwards, we shared the organizational load.

The 1991 Conference celebrated Mitchell's 70th birthday, and an "A.R. Mitchell lecture" was established. The inaugural lecture was given by G. H. Golub, and he is pictured below at the 2005 Conference dinner, along with L. N. Trefethen (A. R. Mitchell lecturer in 2005 and after-dinner speaker in 2007). Golub and Collatz were both given honorary degrees by the University of Dundee.

L. N. Trefethen (left) and G. H. Golub at the 2005 Conference dinner.

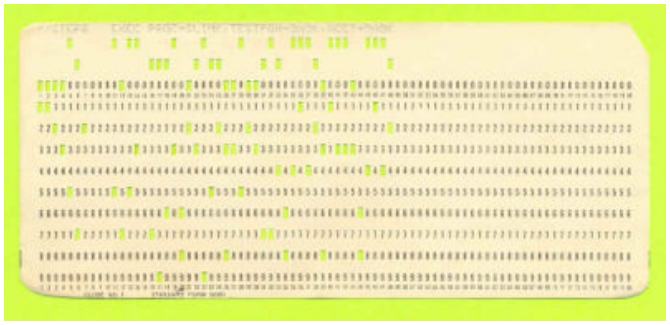
7 The evolution of computing facilities in Dundee

Let me digress a little to say something about the tools which we as numerical analysts used, and the changes to computing facilities in Dundee, most of which I experienced. Of course, corresponding changes occurred in other places.

The first computer in Dundee was a Stantec Zebra, which was purchased (for £13,000) from College funds and private donations in 1961. Numerical results in the Mitchell and Pearce papers were obtained on this machine. In charge was A. J. Cole, who did his PhD in number theory, and had been appointed as a Lecturer in Mathematics in Dundee. He subsequently went on to establish Computer Science in St Andrews (part of Computer Science is now housed in the Jack Cole building, a new building which was named after him in March 2005). The Zebra was a valve machine and input and output was on paper tape. Apparently the start-up procedure involved some banging with fists to ensure that the valves were all properly seated. There was no permanent storage and the memory

consisted of 8K words kept on a magnetic drum. It was about as powerful as a 1990s pocket calculator, though a lot slower.

The first proper Computing Laboratory in Dundee was established in 1965 with J. M. Rushforth as Director and one other member of staff. Rushforth had been appointed a Lecturer in Mathematics in 1953, and took over from Cole in looking after the Stantec Zebra in 1962. (He remained as Director until his retirement in 1992.) In 1965, the Flowers report was published (A Report of a Joint Working Group on Computers for Research). In Scotland, Edinburgh was proposed as a regional centre. Replacements or upgrades for machines then being used (such as the Stantec Zebra in Dundee, Sirius machines at Strathclyde and Heriot-Watt, a KDF9 at Glasgow and an Elliott 803 in Aberdeen), were proposed. The recommendation for Dundee was an ICL machine, but Dundee did not like this proposal, and asked for and got an Elliott 4130 which was installed in 1967. This machine had 32K memory, 2 μsec access time, 48 bit word, magnetic tapes and Algol and Fortran as high level languages. Programmes were punched onto cards and run as batch jobs. It took about one hour every morning to "boot up " the machine and to carry out testing of store, card reader, half-inch magnetic tape, etc.



Punched card.

Card punch machine

About 1971, the Kent Operating System provided on-line remote programming to the

Elliott 4130 using BASIC. Teletypes laboriously printed at about 110 Baud (10 characters per second). A Modular One minicomputer was added as a front-end processor for remote access which by 1974 allowed dial-up modems at 2400 Baud. The Elliott 4130 was replaced in 1977 by a DEC System 10. Microcomputers arrived in 1979 (the BBC micro was about as powerful at the Elliott 4130), and around this time, email became possible to a limited range of contacts. The campus was wired for remote access to the DEC10. Mathematics acquired (black and white) VDUs (Visual Display Units), some of which could work at 4800 Baud, while others only managed 1200 Baud.

In 1984 JANET (Joint Academic Network) was created, which connected about 50 sites in the UK. Terminal access became available for logging into campus and national computers, for file transfer and for email. Colour screens on micros came in about 1985. In 1987 the DEC 10 in Dundee was replaced by a Prime 6350, and an Alliant miniSupercomputer was added the following year, the first true UNIX system.

But distributed systems had started to compete with large mainframe computers. Around 1987, members of the Department of Mathematics and Computer Science (as it was then) in Dundee acquired a large number of SUN Workstations. A SUN 3/160 file server was installed first and there was progressive expansion to SUN3s, then SUN4s. Email still required to use designated gateways: there was a convoluted system where each day's email was downloaded to the Prime. However, by the end of the 1980s, links became available to most networks in the world. Initially, the UK and the US adopted different address formats, which caused some problems, but this was eventually standardized to the US system.

Up to the mid 1980s, it was customary for secretaries to type papers and documents. The advent of document processing systems changed much of that. I remember first using nroff and troff (developed for the Unix operating system) to produce papers, before LaTex became universal in the numerical analysis community.

By 1990, things had moved a long way towards the kind of facilities which we take for granted today.

8 Postscript

My intention here has been primarily to try to give a systematic account of the development of numerical analysis in Scotland from my base year of 1913. I have chosen to take a particular route, which I judge to be the main highway. However, in sticking to this, I have inevitably passed many side roads without stopping, and I should not leave the impression that numerical analysis was only carried out in those places in Scotland so far mentioned.

For example L. F. Richardson (1881– 1953), who is well known for his work on finite differences for solving differential equations, was an Englishman, did much of his work in England, but spent time in Scotland. From 1913 to 1918, he worked for the Meteorological Office as Superintendent of the Eskdalemuir Observatory, located in the south of Scotland. Later on, he spent the period from 1929–1940 as Principal of Paisley College of Technology and School of Art. He retired in 1940 so that he could concentrate on his research.

Ben Noble (1922–2006) was born near Aberdeen in Scotland. After graduating from the University of Aberdeen in radiophysics, he (like some others mentioned in this article) worked at the Admiralty Research Laboratory. After the war he obtained a master's

degree from Cambridge, where he was influenced by lectures from Hartree, and stayed on at the newly established Cambridge Mathematical Laboratory. Following a spell working for the Anglo-Iranian Oil Company (now BP) and three years at the University of Keele, he spent the period from 1955 to 1962 at the Royal College of Science and Technology in Glasgow where he continued to teach while doing a DSc at the University of Aberdeen. He published two books on numerical analysis shortly after this [11], [12] in the Oliver and Boyd University Mathematical Texts Series, edited by Aitken and Rutherford. He moved to the USA in 1962, eventually succeeding J. B. Rosser as director of the Mathematics Research Center at the University of Wisconsin-Madison.

In 1964, the Royal College of Science and Technology became the University of Strathclyde. A Chair of Numerical Analysis was established in 1966 (one year before Dundee), and this was held by D. S. Butler till his retirement in 1994. D. M. Sloan, who joined the staff of Strathclyde in 1965, succeeded Butler, and was instrumental in building up numerical analysis in that institution.

In fact, there have been significant changes in the Scottish scene since the 1990s, precipitated mainly by alterations to University funding arrangements. Dundee lost Research Council funding for its Numerical Analysis MSc course, and although it continued to run for a few years for self-funded students, University pressure to rationalize courses with small numbers contributed to its closure in 1997. Those Mathematics Departments with secure income streams based on large undergraduate student numbers were well placed for growth, and other Universities in Scotland started to build up numerical analysis groups, in particular Strathclyde, as already mentioned, but also Edinburgh and Heriot-Watt.

D. B. Duncan did a PhD at Dundee and after spending some time in Canada moved to Heriot-Watt in 1986. He and Sloan were responsible for the organization of a one day meeting held at Strathclyde University in 1992 ("The Scottish Computational Mathematics Symposium"), whose stated aim was to bring together mathematicians and others who develop computer algorithms to solve mathematical problems. This continues as an annual event, normally with meetings alternating between Strathclyde and Heriot-Watt. The final Dundee biennial conference was held in 2007, ending a 42 year span of such meetings, although it is intended to continue the series from 2009 at Strathclyde. There are now as many numerical analysts at Strathclyde as mathematicians (of all kinds) in Dundee. Indeed Strathclyde currently has one of the largest and most diverse numerical analysis groups in the UK, and the reality is that there has over the last decade or so been a shift in the centre of gravity of numerical analysis in Scotland.

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(http://www.chilton-computing.org.uk/acl/technology/atlas/p009.htm).

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